3.7 Determine the radii of curvature for the trajectories of air parcels located 500 km to the east, north, south and west of the center of a circular low-pressure system, respectively. The system is moving eastward at 15 m s\(^{-1}\). Assume geostrophic flow with a uniform tangential wind speed of 15 m s\(^{-1}\).

**Answer**

![Figure 1](image)

**Figure 1** Definition sketch of the points, having geostrophic windspeed V around a low-pressure system that moves eastward with speed C.

Use Holton Eq. (3.24):

\[
R_t = R_s \left( 1 - \frac{C \cos \gamma}{V} \right)^{-1}
\]

**East:** 
\[
R_t = 500 \times \left( 1 - \frac{15 \times \cos 90^\circ}{15} \right)^{-1} = 500 \text{ km}
\]

**North:** 
\[
R_t = 500 \times \left( 1 - \frac{15 \times \cos 180^\circ}{15} \right)^{-1} = 250 \text{ km}
\]

**West:** 
\[
R_t = 500 \times \left( 1 - \frac{15 \times \cos 270^\circ}{15} \right)^{-1} = 500 \text{ km}
\]

**South:** 
\[
R_t = 500 \times \left( 1 - \frac{15 \times \cos 0^\circ}{15} \right)^{-1} = \infty \text{ km}
\]

3.10 The mean temperature in the layer between 750 and 500 hPa decreases eastward by 3° per 100 km. If the 750-hPa geostrophic wind is from the southeast at 20 ms\(^{-1}\), what is the geostrophic wind speed and direction at 500 hPa? Let \( f = 10^{-4} \text{ s}^{-1} \).

**Answer**

We start with the thermal wind relations (3.32):

\[
u_r = -\frac{R}{f} \left( \frac{\partial T}{\partial y} \right)_p \ln \left( \frac{p_0}{p_1} \right) = 0
\]
\[ v_T = + \frac{R}{f} \left( \frac{\partial T}{\partial x} \right)_p \ln \left( \frac{p_0}{p_1} \right) = \frac{287}{10^{-4}} \times \left( \frac{-3}{100 \times 10^3} \right) \times \ln \left( \frac{75}{50} \right) = -34.9 \text{ ms}^{-1} \]

Now that we know the direction and speed of the thermal wind (see Figure 2) it is possible to calculate the geostrophic wind at 500 hPa. In the triangle formed by all three wind speeds we can use the cosine rule to calculate \( v_{g,500} \):

\[ v_{g,500}^2 = 34.9^2 + 20^2 - 2 \times 34.9 \times 20 \times \cos 45^\circ = 25.2 \text{ ms}^{-1} \]

For the direction of \( v_{g,500} \) we apply the sine-rule in the same triangle:

\[ \frac{\sin \alpha}{20} = \frac{\sin 45^\circ}{25.2} \Rightarrow \alpha = 34^\circ \]

**3.11 What is the mean temperature advection in the 750- to 500-hPa layer in Problem 3.10?**

**Answer**

See Figure 2: the temperature advection is given by: \( -V \cdot \nabla T \). It does not really matter which wind speed we take: either \( v_{g,750} \) or \( v_{g,500} \):

\[ -V \cdot \nabla T = -V_{g,500} \cdot \nabla T = -25.2 \times \cos 56^\circ \times 3 \times 10^{-5} = -4.2 \times 10^{-4} \text{C}^{-1} \text{h} \]

\[ = -V_{g,750} \cdot \nabla T = -20 \times \cos 45^\circ \times 3 \times 10^{-5} = -4.2 \times 10^{-4} \text{C}^{-1} \text{h} \]

**Figure 2** Wind speeds (arrows) and temperature gradient directions.