Notes

The Ageostrophic Wind

1. Definition

If the flow is not “balanced” then the real wind (**V**) deviates from the geostrophic wind ($\mathbf{V_G}$). This deviation is called the **ageostrophic wind** ($\mathbf{V_A}$) for which we can write (see Figure 1):

$$\vec{V}_A = \vec{V} - \vec{V}_G \tag{1}$$

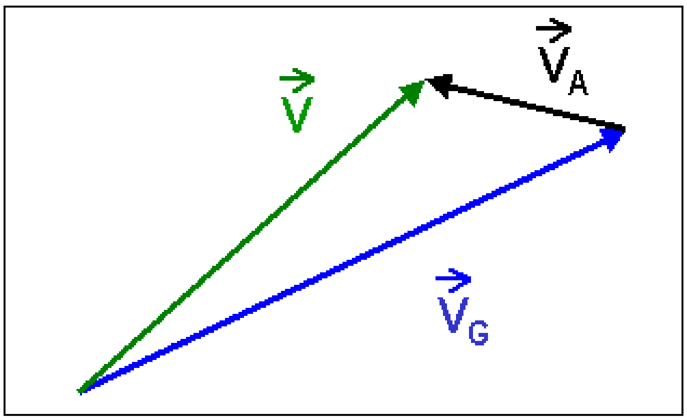

Figure 1. Definition sketch of the ageostrophic wind ($\mathbf{V_A}$).

Usually we have that $|\mathbf{V_A}| << |\mathbf{V_G}|$, with approximately $|\mathbf{V_A}| / |\mathbf{V_G}| = 1/10 = R_O$. In components Eq. (1) gives:

$$u \quad = \quad u_G + u_A \quad \text{and} \quad v \quad = \quad v_G + v_A. \tag{2}$$

From the simplified equations of motion (Holton 2.24 and 2.25) we can write then:

$$\frac{Du}{Dt} \quad = \quad f(v - v_G) \quad = \quad fv_A \tag{3a}$$

$$\frac{Dv}{Dt} \quad = \quad -f(u - u_G) \quad = \quad -fu_A \tag{3b}$$

which simply leads to:

$$u_A \quad = \quad -\frac{1}{f}\frac{Dv}{Dt} \tag{4a}$$

$$v_A = \frac{1}{f}\frac{Du}{Dt}. \qquad (4b)$$

In vectorial form Eqs. (4a) and (4b) read:

$$\vec{V}_A = \frac{1}{f}\vec{k}\times\frac{D\vec{V}}{Dt} \approx \frac{1}{f}\vec{k}\times\frac{D\vec{V}_G}{Dt}. \qquad (5)$$

From these expressions we can see that:

- the ageostrophic wind is a measure of the horizontal acceleration, and
- the ageostrophic wind is perpendicular to the acceleration vector and directed to the left on the northern hemisphere.

2. Defining the isallobaric wind

As an approximation to Eq. (4a), and expanding the *D/Dt* operator, we find:

$$u_A \approx -\frac{1}{f}\frac{Dv_G}{Dt} = -\frac{1}{f}\frac{\partial v_G}{\partial t} - \frac{1}{f}\left(u\frac{\partial v_G}{\partial x} + v\frac{\partial v_G}{\partial y}\right)$$

substituting in this equation the geostrophic relation:

$$v_G = \frac{1}{\rho f}\frac{\partial p}{\partial x}$$

we find:

$$u_A = -\frac{1}{\rho f^2}\frac{\partial}{\partial x}\left(\frac{\partial p}{\partial t}\right) - \frac{1}{f}\left(u\frac{\partial v_G}{\partial x} + v\frac{\partial v_G}{\partial y}\right) \qquad (6a)$$

and, analogous

$$v_A = -\frac{1}{\rho f^2}\frac{\partial}{\partial y}\left(\frac{\partial p}{\partial t}\right) + \frac{1}{f}\left(u\frac{\partial u_G}{\partial x} + v\frac{\partial u_G}{\partial y}\right). \qquad (6b)$$

The first term on the rhs. of Eqs. (6a) and (6b) is the **isallobaric wind (V_I)**:

$$u_I = -\frac{1}{\rho f^2}\frac{\partial}{\partial x}\left(\frac{\partial p}{\partial t}\right), \qquad v_I = -\frac{1}{\rho f^2}\frac{\partial}{\partial y}\left(\frac{\partial p}{\partial t}\right). \qquad (7)$$

The second term on the rhs. of Eqs. (6a) and (6b) is what is called the **diffluence effect** (**V_D**):

$$u_D = -\frac{1}{f}\left(u\frac{\partial v_G}{\partial x} + v\frac{\partial v_G}{\partial y}\right) \quad (8a)$$

$$v_D = +\frac{1}{f}\left(u\frac{\partial u_G}{\partial x} + v\frac{\partial u_G}{\partial y}\right). \quad (8b)$$

3. Sample applications

3.1 Isallobaric wind and isallobars

From Eq. (7) it follows that the isallobaric wind is determined by the gradient of the isolines of (∂p/∂t), these are lines connecting point with the same amount of surface pressure change: the **isallobars**. The direction of the isallobaric wind is perpendicular to the isallobars.

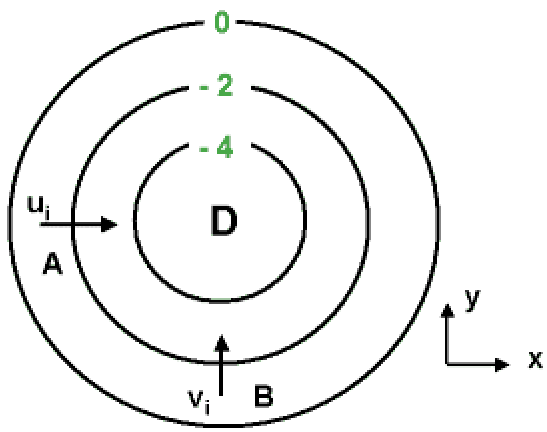

Figure 2. Isallobars and the direction of the isallobaric wind (see text).

In Figure 2 in point A we have: $\frac{\partial}{\partial x}\left(\frac{\partial p}{\partial t}\right) < 0$ hence, from Eq. (7): $u_I > 0$. In point B we have $\frac{\partial}{\partial y}\left(\frac{\partial p}{\partial t}\right) < 0$ resulting in $v_I > 0$. From this we can conclude that the isallobaric wind always points to the minimum value where the strongest decrease in surface pressure is located.

3.2 General observation of isallobaric wind

From Figure 3 (next page) it is obvious that, as usual, $\boldsymbol{V_G}$ is aligned along the isobars (with the lowest pressure on the left). And ∂**V**/∂t is aligned with the isallobars with the minimum (maximum decrease) on the left. The isallobaric wind itself is perpendicular to the isallobars and hence also perpendicular to the acceleration (pointing to the left).

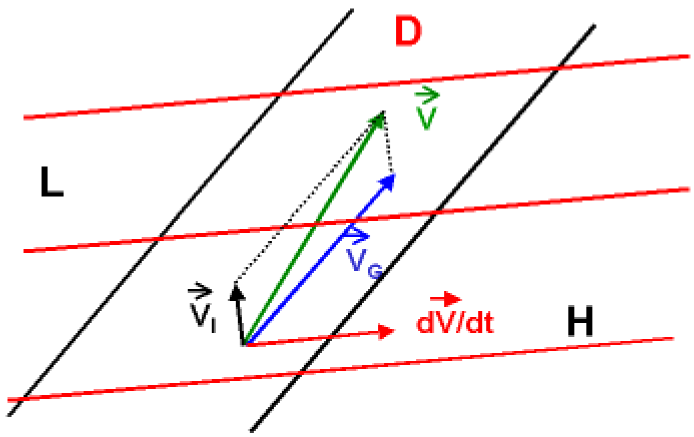

Figure 3. Isobars (black) and isallobars (red) and the definition of the various winddirections. L: low pressure, H: high pressure D maximum decrease in surface pressure.

3.3 Diffluence (and confluence) effects.

From Eqs. (8a) and (8b) we can calculate the direction of the components of the diffluent wind, when the geostrophic wind is not uniform. In Figure 4 the simplest case of a diffluent flow is given.

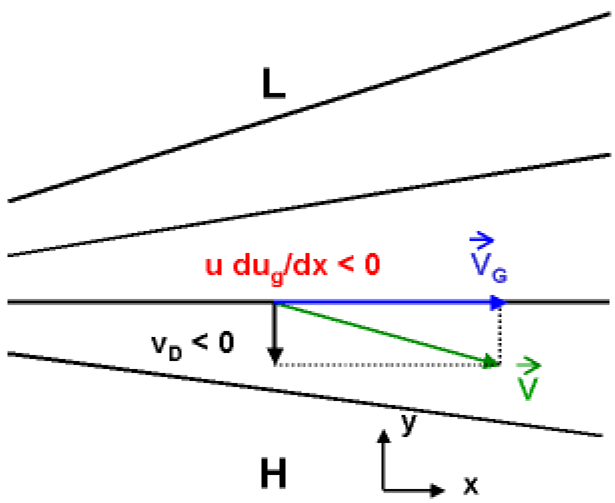

Figure 4. Isobars (black lines) in a diffluent flow pattern and the direction of the various wind components.

In the case depicted the geostrophic wind will decrease in the positive x-direction due to the larger distance between the isobars and the consequently smaller pressure gradient. As $u > 0$ then we have

$$u\frac{\partial u_G}{\partial x} < 0$$

and hence, from Eq. (8b) $v_D < 0$. In this case the windspeed will decrease as the ageostrophic component is pointing in the direction of higher pressure. In the analogous case of a confluent flow (not depicted here) the windspeed will increase.